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SHORTER COMMUNICATIONS

DIRECT CONTACT HEATING OF LAMINAR FALLING LIQUID JETS

N. S. MURTY* and V. M. K. SASTRI†
Indian Institute of Technology, Madras, India

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NOMENCLATURE

C_p,	specific heat of the liquid;
g,	acceleration due to gravity;
h_{fg},	latent heat of condensation;
h_x,	local heat-transfer coefficient;
K,	ratio of viscosities (μ/μ_v);
K_0,	thermal conductivity of the liquid;
Pr,	Prandtl number $(\mu C_p/K_0)$;
Re,	Reynolds number $(\rho U_0/h_0/\mu)$;
t,	temperature;
ΔT,	temperature difference $(t_s - t_0)$;
u, U,	velocity components in the flow direction;
U_0,	maximum incoming velocity in the nozzle;
U_δ,	interface velocity;
x, r,	coordinates;
x_d,	space coordinate from where the flow is thermally fully developed;
$\bar{x}_d, \bar{x}$,	(x_d/h_0), (x/h_0).

Greek symbols

α,	$(\rho g h_0^2/\mu U_0)$;
β,	$(C_p \Delta T/h_{fg})$;
γ,	$(\rho\mu/\rho_v \mu_v)$;
$\delta, \bar{\delta}$,	radius of the jet, (δ/h_0);
δ_v,	radius of the vapour boundary layer;
δ_t,	thermal boundary-layer thickness;
λ,	dimensionless coordinate $(\bar{x}/Re)$;
ρ, ρ_v,	densities of liquid and vapour;
μ, μ_v,	viscosities of liquid and vapour;
ξ, ξ_v,	(r/δ), $(\psi_v - \xi)/(\psi_v - 1)$;
ψ_t, ψ_v,	(δ_t/δ), (δ_v/δ).

DIRECT contact heating finds applications mainly in regenerative feed water heaters, deaerators and evaporators. The advantages of simple design, attainable high heat-transfer coefficients and total absence of scale formation attract the attention of the researchers [1–6]. Also due to their trouble-free and high performance efficiency they can be used effectively for low pressure regenerative feed water heaters for large capacity power stations [7] and combined power and fresh water plants [8]. It appears that direct contact heating of accelerating liquid jets, a problem approximating the heating of feed water between two trays of a deaerator and direct contact heater, has not been investigated and hence an attempt is made here to study the same analytically with suitable assumptions.

MATHEMATICAL FORMULATION

The physical model is shown in the Fig. 1. In this analysis the static pressure variation, surface tension, buoyancy effects and surface resistance are assumed to be negligible. It is also assumed that the physical properties of the liquid are temperature independent and vapour and thermal boundary layers develop in the vapour region and liquid jet from the interface at $x = 0$. The thermal boundary-layer thickness increases till it reaches the axis of the liquid jet at $x = x_d$. This region is named as thermally developing and the one beyond is thermally developed. With these assumptions the steady laminar equations for mass, momentum and energy can be written in the integral form as follows:

(A) *Thermally developing region*

$$\frac{d}{dx}\int_0^\delta \rho u r\,dr = \dot{m} \tag{1}$$

$$\frac{d}{dx}\int_0^\delta \rho u^2 r\,dr - U_\delta \dot{m} = \rho g \delta^2/2 + \mu\left(r\frac{\partial u}{\partial r}\right)\Big|_0^\delta \tag{2}$$

$$\frac{d}{dx}\int_\delta^{\delta_v} \rho_v U^2 r\,dr + U_\delta \dot{m} = \mu_v\left(r\frac{\partial U}{\partial r}\right)\Big|_\delta^{\delta_v} \tag{3}$$

$$\frac{d}{dx}\int_{\delta-\delta_t}^\delta \rho u C_p t r\,dr - C_p t_s \dot{m} + C_p t_0 \frac{d}{dx}\int_0^{\delta-\delta_t} \rho u r\,dr = K_0\left(r\frac{\partial t}{\partial r}\right)\Big|_{\delta-\delta_t}^\delta. \tag{4}$$

The appropriate boundary conditions with respect to the physical model can be written as

at $\quad x = 0 \quad u = U_0(1 - r^2/h_0^2) \qquad (5)$

$$r = 0 \quad \frac{\partial u}{\partial r} = 0 \tag{6}$$

$$r = \delta - \delta_t \quad t = t_0;\ \frac{\partial t}{\partial r} = 0 \tag{7}$$

$$r = \delta \quad u = U = U_\delta;\ \frac{\partial U}{\partial r} = \frac{\partial u}{\partial r};\ t = t_s \tag{8}$$

$$h_{fg}\dot{m} = K_0 \delta \frac{\partial t}{\partial r} \tag{9}$$

$$r = \delta_v \quad U = 0;\ \frac{\partial U}{\partial r} = 0. \tag{10}$$

To solve these integral relations equations (1)–(4) together with the boundary conditions equations (5)–(10); velocity and temperature profiles are assumed to be

$$u = U_0(a_0 - a_1 \xi^2) \tag{11}$$

$$U = U_\delta \xi_v^2 \tag{12}$$

$$T_1 = \frac{(t - t_0)}{\Delta T} = \xi_t^2 \tag{13}$$

where

$$\xi_t = \frac{(\xi + \psi_t - 1)}{\psi_t}. \tag{14}$$

(B) *Thermally developed region*

In this region all the governing integral equations (1)–(4) are valid except equation (4) which takes the form

$$\frac{d}{dx}\int_0^\delta \rho u C_p t r\,dr - C_p t_s \dot{m} = K_0\left(r\frac{\partial t}{\partial r}\right)\Big|_0^\delta \tag{15}$$

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*Presently Development Engineer, BHEL (R & D) Hyderabad, India.
†Associate Professor, Department of Mechanical Engineering, IIT, Madras, India.

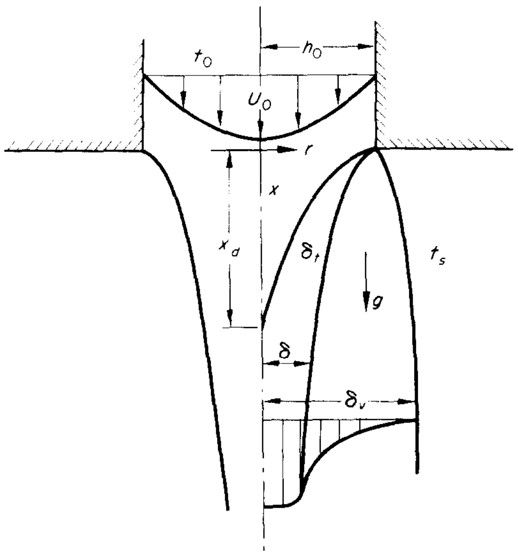

FIG. 1. Physical model.

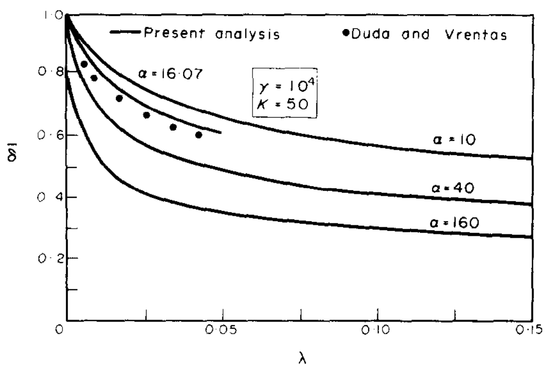

FIG. 2. Jet radius.

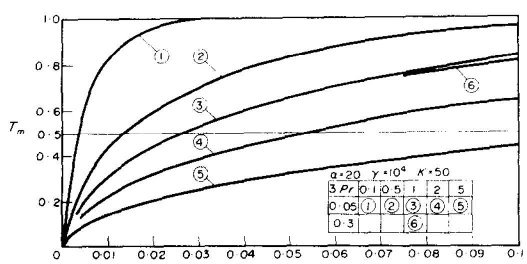

FIG. 3. Mean temperature.

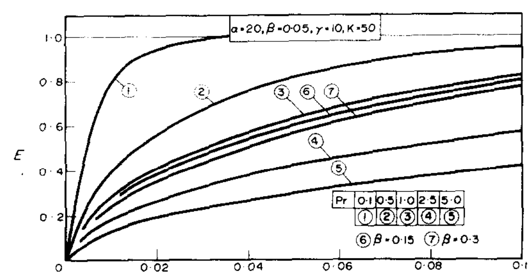

FIG. 4. Effectiveness.

and the boundary conditions given in equations (5)–(10) are to be satisfied with the exception of equation (7) and hence the velocity profiles as selected in the previous section are still valid but the temperature profile is to be modified for this region as

$$T_2 = \frac{t - t_0}{\Delta T} = \phi + (1-\phi)\xi^2 \tag{16}$$

where ϕ is an arbitrary function of $\bar{x}$ and is so selected that

$$\text{at } x = x_d \quad T_1 \equiv T_2. \tag{17}$$

This gives the initial condition for ϕ as

$$\phi(\bar{x}_d) = 0. \tag{18}$$

Substitution of the selected velocity and temperature profiles into the governing equations results in a set of first order ordinary differential equations. Modified Euler's integration technique is utilized for the solution and all the calculations are carried out on an IBM 370/155 digital computer to an accuracy of 10^{-4}.

HEAT-TRANSFER EXPRESSIONS

(a) *Mean temperature*

Defining the mean temperature in the liquid jet as

$$t_m = \frac{1}{\pi\delta^2}\int_0^\delta 2\pi rt\,\mathrm{d}r. \tag{19}$$

It can be shown for thermally developing region that

$$T_m = \frac{t_m - t_0}{\Delta T} = 2\psi_t(1 - 0{\cdot}25\psi_t)/3 \tag{20}$$

and for thermally developed region that

$$T_m = \frac{t_m - t_0}{\Delta T} = (1+\phi)/2. \tag{21}$$

(b) *Effectiveness*

Defining the effectiveness E as the ratio of condensate to the maximum possible condensate for the given rate of liquid flow as

$$E = \frac{\int_0^\delta 2\pi r\rho u\,\mathrm{d}r - \int_0^\delta 2\pi r\rho u\,\mathrm{d}r\Big|_{x=0}}{\beta\int_0^\delta 2\pi r\rho u\,\mathrm{d}r\Big|_{x=0}} \tag{22}$$

it can be shown after simplification, that

$$F = [\delta^2(2a_0 - a_1) - 1]/\beta. \tag{23}$$

RESULTS AND DISCUSSION

The dimensionless liquid jet radius for the case of no condensation and $\alpha = 16{\cdot}07$ is compared to the experimental results of Duda and Vrentas [9] in Fig. 2. Though the present analysis is an approximate one, fair agreement is observed downstream except very near to the entrance where the maximum deviation is found to be about 10 per cent. The variation of the dimensionless mean temperature is shown in Fig. 3. It is noted that the dimensionless mean temperature attains rapidly its limiting value of unity for small Prandtl number. This is due to the fact that heat capacity of the liquid is small for low Prandtl numbers. It is found that the influence of the parameter α on the dimensionless mean temperature is not significant. This can be attributed to the fact that with an increase in α, the jet radius and, consequently, the heat-transfer surface decrease with one countering the effect of the other. From equation (20) it can be seen that the dimensionless mean temperature assumes a value of 0·5 at the point where the developing thermal boundary-layer touches the axis of the liquid jet.

Hence the line $T_m = 0{\cdot}5$ separates the thermally developing and developed regions.

The effectiveness is plotted in Fig. 4 against the dimensionless distance along the flow direction. It is seen that it rapidly approaches its limiting value of unity for small Prandtl numbers. From the same figure it can be noted that the influence of β is significant compared to that of Prandtl number by observing the curves 6 and 7 which are plotted for $Pr = 1$. It is found that the effectiveness is a weak function of α as was observed for dimensionless mean temperature.

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FILM COOLING IN ADVERSE PRESSURE GRADIENTS

GEORGE W. HAERING
Escuela Politécnica del Litoral, Apartado 5863, Guayaquil, Ecuador

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NOMENCLATURE

E, the film effectiveness, wall–main stream temperature difference/wall–main stream temperature difference at injection slot [dimensionless];
X, the distance downstream of the slot [in];
S, the height of the slot opening [in];
M, the ratio of the mass velocities, the coolant's divided by the main stream's [dimensionless];
A, dimensionless factor accounting for the effects of geometry, turbulence and boundary layers;
θ, momentum thickness of the main stream at the slot location [in].

IN THIS brief note, film cooling will refer to the process of injecting a gas along a wall, through discrete openings, to shield the wall from a high temperature gas stream. A "film" of gas persists for a distance downstream of injection, its effect gradually eroded by mixing with the main stream. Figure 1 gives an idea of the geometry involved.

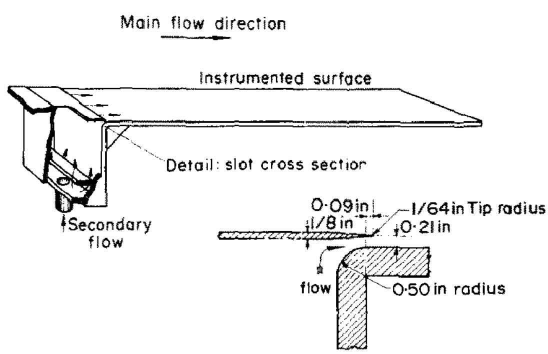

FIG. 1. The film cooling injection system.

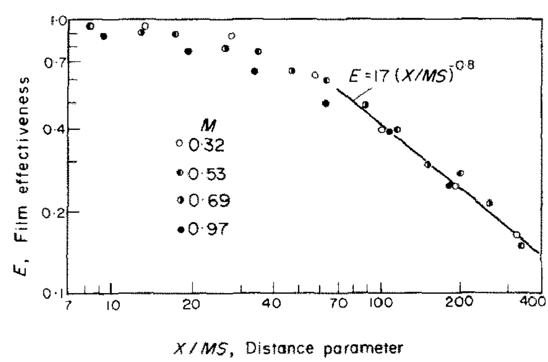

FIG. 2. Experimental results without pressure gradient.

The process can insulate effectively in high heat flux situations, and has also been proposed to control boundary layer separation in an adverse pressure gradient. Hypersonic airbreathing engines could use these traits, to name a possible application.

Pai and Whitelaw [1] presented the important step of an analytical method (a computer solution of the boundary-layer equations with a mixing length concept) for the prediction of film cooling effectiveness. The method predicts a quite small effect for adverse pressure gradients. Escudier and Whitelaw [2] found experimentally a small pressure gradient effect for the introduction of coolant through a porous plug rather than a slot. Several experimental studies with favorable pressure gradients present a not entirely consistent view, but an indication that the effect is also small.

Experiments were performed to investigate film cooling with adverse pressure gradients, in which air at moderate pressure, temperature and velocity flows through a duct of rectangular cross section. The inside vertical dimension is constant; the inside horizontal dimension is variable. Slots